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APPLICATION OF DIFFERENTIAL EQUATIONS IN ECONOMIC PROBLEMS

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A R T I C L E I N F O.

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Abstract

This paper considers the application of differential equations in some problems of economic processes. Various economic issues lead to the need to solve equations containing as an unknown a certain function and its derivatives up to a certain order n.

Differential equations are used to mathematically describe natural phenomena. Differential equations are also widely used in models of economic dynamics. These models reflect not only the dependence of variables on time, but also their relationship over time.

To study economic processes, scientists use methods of mathematical analysis, linear programming, matrix and vector calculus. The basis of economic theory consists of economic laws expressed in the form of quantitative relations between the quantities characterizing the economic system or process. Such laws make it possible to study real economic systems based on mathematical models. The construction and study of these models is the subject of mathematical economics, which considers the economy as a complex dynamic system. [2.64]

To study mathematical models of economics, in addition to economics, it is necessary to master mathematical methods, among which the apparatus of differential equations plays an important role. Economic patterns, as a rule, are complex nonlinear relationships between economic quantities, the explicit form of which is difficult to establish directly. In the presence of a stable pattern, small changes in values can be approximately replaced by differentials. Then the nonlinear relations between the quantities, respectively, are replaced by simpler linear relations between the quantities and their derivatives. These relations are differential equations that are used to construct a mathematical model of an economic system or process. For example, models of economic processes based on differential equations. Various economic issues lead to the need to solve equations containing as an unknown a certain function and its derivatives up to a certain order *n*.

Task1. An economic problem leading to a differential equation.

Let's denote K - the amount of funds in kind or in value. Funds are machines, premises and the like, they wear out, age, as they say, they are being disposed of. The retirement rate of funds is a derivative $\frac{dK}{dt}$, it is expressed in terms of the disposal coefficient μ. For example, if the funds are completely renewed over 10 years, then the retirement ratio is equal to $\mu = \frac{1}{10}$ .On the other hand, investments I – investment of money-leaders to an increase in funds with a proportionality coefficient ρ . Considering all this, we obtain a differential equation of the form:

$$\frac{dK}{dt} = -\mu K + \rho I \ .$$

Task2. Build a model of natural growth (growth at a constant rate).

Solution: Let's $y(t)$ denote the intensity of output of some enterprise (industry). We will assume that there is an axiom about the unsaturation of the consumer, i.e. that all the released goods will be sold, and also that the volume of sales is not so high as to significantly affect the price of the goods p , which we will consider fixed. To increase the intensity of the release $y(t)$, it is necessary that the net investment $I(t)$ (i.e., the difference between the total investment and amortized costs) was greater than zero. In the case $I(t) = 0$

total investments only cover depreciation costs, and the level of output remains unchanged. Case $I(t) < 0$ it leads to a decrease in fixed assets and, as a consequence, to a decrease in the level of output.

Thus, the rate of increase in the intensity of output is an increasing function of

$I(t)$. Let this dependence be expressed by direct proportionality, i.e. the so-called acceleration principle takes place

$$y' = mI \ (m - const),$$

Where $1/m$ acceleration rate

Let α - the net investment rate, i.e. the part of income that is spent on net investments, then $I = \alpha\, py$.

Labeling $k = m\alpha\, p = const$, finally we get the differential equation:

$$y' = ky.$$

Integrating this differential equation with separable variables, we find its general solution:

$$y = Ce^{kt}.$$

Under the initial condition $y(t_0) = y_0$, we will find a particular solution $y = y_0 e^{k(t-t_0)}$.

This solution is called the natural growth equation. It describes the dynamics of price growth at a constant rate of inflation.

Task 3. The Samuelson equation. Web-based market model

Consider the Samuelson equation

$$p' = k(d(p) - s(p)),$$

modeling the relationship between price changes p and unsatisfied demand $d(p) - s(p)$, where $d(p), s(p)$ - accordingly , the values of supply and demand at the price p, $k > 0$. Suppose that supply and demand are given by linear functions

$$d(p) = a - bp,\ s(p) == m + np\ ,$$

where a, b, m, n- some positive numbers.

With this in mind, the differential equation will take the form

$p' = k(n-b)p + k(a+m)$.

This equation is a linear inhomogeneous differential equation and solving we get a general solution:

$p = \dfrac{a-m}{b+n} + Ce^{k(n-b)t}$.

This dependence shows that when $n > b$ over time, the function p will be separated from the equilibrium state $\overline{p} = \dfrac{a-m}{b+n}$. If $n = b$, then it is p - constant, and if $n < b$, then over time p it will asymptotically approach the state of equilibrium

$\overline{p} = \dfrac{a-m}{b+n}$.

This model is considered as a continuous analogue of the web model of the market.

Task 4. Find a function having a constant elasticity equal to *k*. [4, 15].

Solution: By definition, the elasticity of the function is

$E_{xy} = y'\frac{x}{y}$,

then by the condition of the problem we get:

$$y'\frac{x}{y} = k$$

differential equation with separable variables: $\frac{dy}{y} = \frac{dx}{x}k$ integrating both parts we get: $\ln y = \text{k}\ln x + \ln c$ $\;y = x^k c$

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