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GYOLDER SHARTINI QANOATLANTIRUVCHI FUNKSIYALAR SINFI

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ARTICLEINFO.

Tayanch iboralar: Silliq yopiq chiziq, analitik funksiya, tashqi va ichki soxa, uzluksiz funksiya, Geyolder sharti, xosmas integral, xosmas integralning bosh qiymati, bo'lakli analitik funksiya.

Annotatsiya

Ushbu maqolada Koshi tipidagi integral va uning muhim xossalari, Koshi tipidagi integralning bosh qiymati tog'risidagi tushunchalarni hosil qilish hamda Geyolder sinfi to'g'risida amaliy masalalarga qo'llashni talqin qilamiz.

φ(τ) funksiya biror Ye to'plamda berilgan bo'lsin. τ va φ(τ) –haqiqiy yoki kompleks bo'lishi mumkin.

Ta'rif. φ(τ) funksiya Ye to'plamda Gyolder (H(λ)) shartini qanaoatlantiradi deyiladi, agarda ∀τ1,τ2 ∈E lar uchun

$$|\varphi(\tau_1)-\varphi(\tau_2)|\le A|\tau_1-\tau_2|^{\lambda} \qquad (1)$$

tengsizlik o'rinli bo'lsa, bunda A va λ- musbat o'zgarmas sonlar. Agar λ>1 bo'lganda, u holda (1) shartdan φ'(t)=0⇒ φ(t)=const bo'ladi. Shuning uchun 0<λ≤1 deb faraz qilamiz.

λ=1 bo'lganda (1) shart Lipshis shartini ifodalaydi.

Agar φ(τ)∈H(λ) bo'lsa, |φ(τ)|∈H(λ) bo'ladi.

Haqiqattan ham, ∀τ1,τ2 ∈E uchun

$$\big||\varphi(\tau_1)|-|\varphi(\tau_2)|\big|\le|\varphi(\tau_1)-\varphi(\tau_2)|\le A|\tau_1-\tau_2|^{\lambda}\Rightarrow|\varphi(\tau)|\in H(\lambda)$$

ekanligi kelib chiqadi.

Agar φ(τ)∈H(λ) bo'lsa, φ(τ)∈H(λ1) (λ1<λ) bo'ladi.

Buning teskarisi o'rinli emas. Demak, kichik λ ga katta klass to'g'ri keladi.

Eng kichik sinf, bu Lipshis shartini qanoatlantiruvchi funksiyalar sinfi bo'lib hisoblanadi.

Agar φ1(τ), φ2(τ) funksiyalar E to'plamda mos ravishda λ1 va λ2 ko'rsatkichlar bilan Gyolder shartini qanoatlantirsa, u holda ularning yig'indisi, ko'paytmasi va $\frac{\varphi_1(\tau)}{\varphi_2(\tau)}$ $(\varphi_2(\tau)\ne 0,\ \forall\tau\in E)$ ham $\lambda=\min\{\lambda_1,\lambda_2\}$ ko'rsatkich bilan Gyolder shartini qanoatlantiradi.

Agar φ(τ) funksiya differensiallanuvchi bo’lsa, u holda bu funksiya Lipshis shartini qanoatlantiradi. Haqiqattan ham

$$\varphi(\tau_1)-\varphi(\tau_2)=\varphi'(c)(\tau_1-\tau_2),$$

$$\forall\tau_1,\tau_2\in E.\ c\in(\tau_1,\tau_2)\Rightarrow|\varphi(\tau_1)-\varphi(\tau_2)|=|\varphi'(c)|\cdot|\tau_1-\tau_2|\le A|\tau_1-\tau_2|.$$

Buning teskarisi o’rinli emas.

Misol. φ(x)=|x| funksiya Lipshis shartini qanaotalantiradi, lekin bu funksiya koordinata boshida hosilaga ega emas, chunki ung hosilasi +1, chap hosilasi –1.

U=u(ς) funksiya biror E1 sohada aniqlangan bo’lib, unda u Gyolder shartini (E(μ) sharti) qanoatlantirsin. f(u) funksiya esa u(ς) funksiyaning qiymatlar to’plamida aniqlangan bo’lib, u H(ν) shartni qanoatlantirsin. U holda F(ς)=f(u(ς)) funksiya ς-o’zgaruvchi bo’yicha H(μ.ν) shartini qanoatlantiradi. Agar ν=1 bo’lsa, u holda F(ς) funksiya H(μ) shartini qanoatlantiradi.

Misol.

1). $\varphi(x)=\sqrt{x}$ funksiya haqiqiy o’qning har bir intervalida $H(\frac{1}{2}),\quad \lambda=\frac{1}{2}$ shartini qanoatlantiradi, agar interval nolni o’z ichida saqlamasa, u holda bu funksiya intervalda analitik bo’ladi. Shuning uchun u Lipshis shartini qanoatlantiradi.

2). $\varphi(x)=\dfrac{1}{\ln x},\quad 0<x\le\dfrac{1}{2},\quad \varphi(0)=0.$ Bu funksiya $0\le x\le\frac{1}{2}$ da uzluksiz. $\lim\limits_{x\to 0}x^{\lambda}\ln x=0\quad\forall\lambda>0$ учун

A va λ qanday bo’lishidan qa’tiy nazar x ning shunday qiymatini ko’rsatish mumkinki:

$$|\varphi(x)-\varphi(0)|=\left|\frac{1}{\ln x}\right|>A\cdot x^{\lambda}\Rightarrow\varphi(x)$$

funksiya qaralayotgan $[0,\frac{1}{2}]$ oraliqda Gyolder shartini qanoatlantirmaydi.

Gyolder sharti tushunchasini ko’p o’zgaruvchili funksiyalarga ham tarqatish mumkin.

Ta’rif. $\varphi(\tau_1,\tau_2,...,\tau_n)$ funksiya biror D to’plamda aniqlangan bo’lsin. $\varphi(\tau_1,\tau_2,...,\tau_n)$ funksiya D to’plamda Gyolder shartini ($H(\mu_1,\mu_2,...,\mu_n)$) shartini qanoatlantiradi deyiladi, agarda

$$\forall(\tau'_1,\tau'_2,...,\tau'_n),(\tau''_1,\tau''_2,...,\tau''_n)\in D$$

lar uchun

$$|\varphi(\tau'_1,\tau'_2,\dots,\tau'_n)-\varphi(\tau''_1,\tau''_2,\dots,\tau''_n)|\le A_1|\tau'_1-\tau''_1|^{\mu_1}+$$

$$+A_2|\tau'_2-\tau''_2|^{\mu_2}+\dots+A_n|\tau'_n-\tau''_n|^{\mu_n} \tag{2}$$

tengsizlik o’rinli bo’lsa, bunda Ai, μi (i=1,n)-musbat o’zgarmas sonlar.

$\mu_i \leq 1$, i=1,2,…,n.

Agar D to'plam chegaralangan to'plam bo'lsa, u holda (1.2.2) ni quyidagicha ham yozish mumkin:

$$\mu = \min\{ \mu_1, \mu_2, ..., \mu_n \}, \quad A = \max\{ A_1, A_2, ..., A_n \}$$

$$|\varphi(\tau'_1, \tau'_2, \ldots, \tau'_n) - \varphi(\tau''_1, \tau''_2, \ldots, \tau''_n) \leq A_1\{|\tau'_1 - \tau''_1|^{\mu} + $$
$$+ |\tau'_2 - \tau''_2|^{\mu} + \ldots + |\tau'_n - \tau''_n|^{\mu}\} \tag{3}$$

Agar $\varphi(\tau_1, \tau_2, ..., \tau_n)$ funksiya $H(\mu_1, \mu_2, ..., \mu_n)$ shartini qanoatlantirsa, u holda

$$|f(t_1, t_2, \ldots, t'_k, t_{k+1}, \ldots, t_n) - |f(t_1, t_2, \ldots, t_{k-1}, t''_k, t_{k+1}, \ldots, t_n)| \leq A_k |t'_k - t''_k| \tag{4}$$
$$k = 1, 2, \ldots, n$$

shartini ham qanoatlantiradi, ya'ni $\varphi(\tau_1, \tau_2, ..., \tau_n)$ funksiya har bir argumenti bo'yicha ham (qolganlariga nisbatan tekis) Gyolder shartini qanoatlantiradi.

$H(\mu_k)$ shartni va aksincha $\varphi(\tau_1, \tau_2, ..., \tau_n)$ funksiya ayrim-ayrim har bir agrumenti bo'yicha $H(\mu_k)$ shartini qanoatlantirsa (qolganlariga nisbatan tekis), $H(\mu_1, \mu_2, ..., \mu_n)$ shartini ham qanoatlantiradi.

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